

Defunctionalized Interpreters for Call-by-Need Evaluation

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Motivation

Formal semantics: why?

- ▶ Understanding linguistic features
- ▶ Proving programs correct or equivalent
- ▶ Proving language properties
- ▶ Proving implementations, analyses or transformations correct

Formal semantics: which kind?

- ▶ Denotational?
- ▶ Operational? Big step? Small step?
- ▶ Axiomatic?

Foundations of this work

Semantic artifacts can be inter-derived mechanically,

and

the inter-derivation is worthwhile:

- ▶ it can yield simpler semantics, and
- ▶ it can yield new semantics.

Here: call by need.

Call-by-need evaluation

- ▶ Demand-driven computation
- ▶ Memoization of intermediate results

Semantics for call-by-need evaluation

- ▶ Store-based:
results are saved in the global store
- ▶ Storeless:
results are saved in the term itself

Syntactic theories of call by need

- ▶ Different opinions — the POPL'95 affair
 - ▶ The call-by-need lambda-calculus by Ariola and Felleisen (JFP'97)
 - ▶ The call-by-need lambda-calculus by Maraist, Odersky and Wadler (JFP'98)

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- ▶ Appearances can be deceiving:
The standard reduction is common to both
- ▶ It is our starting point

Outline

1. The call-by-need λ -calculus
2. Deriving an abstract machine and a natural semantics

The call-by-name λ_{let} -calculus

Terms $\ni T ::= x \mid \lambda x.T \mid T T \mid \text{let } x \text{ be } T \text{ in } T$

Values $\ni V ::= \lambda x.T$

Eval Cont $\ni E ::= [] \mid E T \mid \text{let } x \text{ be } T \text{ in } E$

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Accounting for call by need syntactically

On-demand: already present for call by name

$$\text{let } x \text{ be } T \text{ in } E[x] \rightarrow \text{let } x \text{ be } T \text{ in } E[T]$$

Memoization: restricting substitution to values

$$\text{let } x \text{ be } V \text{ in } E[x] \rightarrow \text{let } x \text{ be } V \text{ in } E[V]$$

and adapting the search

(hence the extra context constructor)

Related semantics for call by need

- ▶ A variety of fascinating semantics exist:
TIM (Fairbairn and Wray, FPCA'87)
Lazy Krivine Machine (Sestoft, JFP'97)
Maraist et al., POPL'98
Garcia et al., POPL'09
etc.
- ▶ How do they relate to the call-by-need λ -calculus?

Our thesis

What: We can constructively calculate the corresponding abstract machine and other semantic artifacts.

Our thesis

What: We can constructively calculate the corresponding abstract machine and other semantic artifacts.

How: We use program transformations off the shelf.

Our sub-thesis

- ▶ Spell out the reduction semantics in detail
- ▶ Leads directly to the abstract machine

The reduction semantics

Decomposition

Searches for the next redex,
if there is one, and its context

Contraction

Contracts a redex to a contractum

Recomposition

Reconstructs a term
from a contractum and the context

The reduction semantics

Decomposition

Searches for the next redex,
if there is one, and its context

Contraction

Contracts a redex to a contractum

Recomposition

Reconstructs a term
from a contractum and the context

Hygiene

We maintain hygiene explicitly
(e.g., a variable convention)

The reduction semantics

One-step reduction

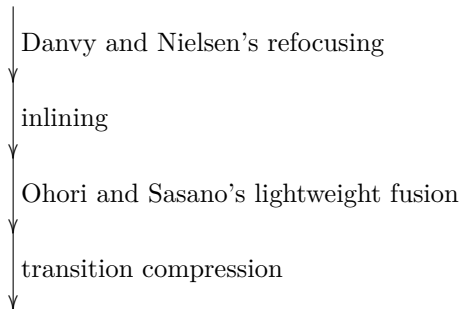
Decompose, contract and recompose

Reduction-based evaluation

Iterate one-step reduction

The syntactic correspondence

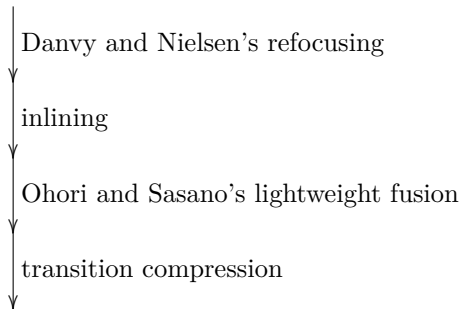
Reduction semantics



Abstract machine

The syntactic correspondence

Reduction semantics

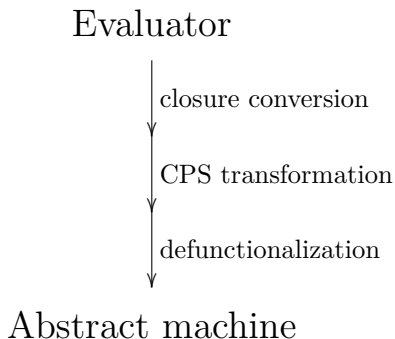


Abstract machine

Result: the simplest storeless abstract machine for call-by-need evaluation (see paper).

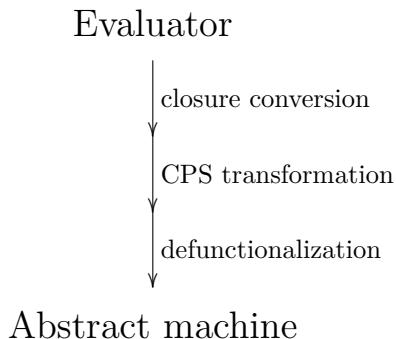
The functional correspondence

Kudos for John Reynolds:



The functional correspondence

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Result: the first storeless natural semantics for call-by-need evaluation (see paper).

Our results

- ▶ Identify what is common to Ariola et al.: the standard reduction
- ▶ Calculate the corresponding abstract machine
- ▶ Calculate the corresponding natural semantics
- ▶ And also: calculate the corresponding extended direct semantics
- ▶ A denotational semantics can be constructed with a control monad or a state monad

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Thank you.